

Optimal Structure Control Design of a Parallel Robot for End-Effector Trajectory Tracking

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Abstract

As the demand for special purpose robotic manipulators has increased, task based design for these manipulators become the key for their optimum performance. Hence, this paper presents a task based integrated design approach (TBIDA) for simultaneously optimizing both the structure design parameters of a parallel robot and model-based controller gains in order to perform a given task. A nonlinear dynamic optimization problem for this approach is stated and it is solved by using a differential evolution technique. A dexterity measure and positioning errors are used as performance indexes. The planar five-bar parallel robot is designed by the TBIDA. The effectiveness of this design methodology is shown via simulation results.

1 Introduction

In the last years, integrated design of both the robotic manipulator structure and its controller has been the approach for improving their design [1]–[4]. Since controllers in industry are not sophisticated, the philosophy in the integrated design approach is to use simple controllers that do not depend on the system model, such as PI, PD and PID controllers. Sudhendu and Asada [1] proposed an integrated design approach. They presented the integrated structure control design of high speed single link robots based on time-optimal control and finite element analysis. With this approach an improved performance of a robotic manipulator design was obtained. Fu and Mills [2] proposed a structure and control design approach based on convex optimization theory, linear model of the system and linear control laws. In this approach, the design specifications (performance indexes) must be convex with respect to the closed-loop transfer function. Ravichandran et al. [3] presented a methodology based on optimization techniques for simultaneously optimizing the design parameters of a two-link planar rigid manipulator and the gains of a nonlinear PD controller, in order to perform multiple tasks. Gravity compensation and position errors were the optimization criteria for the structure and controller design. In Portilla et al. [4] a structure-control integration approach is used in order to design a pinion-rack continuously variable transmission. A multi objective dynamic optimization problem is stated in order to concurrently obtain

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a set of optimal mechanical structure and controller parameters. A kinematic performance index and a dynamic performance index were used as criteria for the integrated design approach.

Generally, the robotic manipulators are used to execute one or a couple of tasks, then a task based design to design an optimal manipulator for given tasks become more suitable. In [7], the link lengths of a parallelogram five-bar parallel robot were optimized in order to fulfill a given task. A task based kinematic design approach for designing a parallel robot is developed. Therefore only the closed-loop kinematic constraints are considered into the optimization design instead of the dynamic system model constraint.

A special case of robotic manipulators are parallel manipulators which have received considerable attention from the research community over the past two decades [5]. Their study is motivated by the fact that they can provide higher stiffness and accuracy than serial ones, especially for a specific task. However, the dynamics and kinematics of parallel manipulators are more complex than those arising in the analysis of serial ones.

Optimization techniques have been used in order to improve the parallel manipulator design by considering kinematic [6] and dynamic [7] performance indexes. Other works have considered dynamic performance criteria, such as dynamic manipulability [7] and dynamic conditioning index [8]. Nevertheless, all of these performance indexes are optimized without considering the temporary constraint of the dynamic system. Then, when the design is for a specific task, the temporary constraint of the dynamic system must be included in order to provide states or data of the system more alike for the real manipulator behavior.

In this paper, a task based integrated design approach (TBIDA) is stated as a nonlinear dynamic optimization problem (NLDOP). In this approach, both the structure and the PID controller gains are simultaneously designed in order to perform a given task. In the problem statement the dynamic manipulability measure [9] and the positioning errors are simultaneously optimized for improving the manipulating ability and the end-effector positioning accuracy of the planar five-bar parallel robot by including both kinematic and dynamic constraints. A differential evolution algorithm for solving constrained optimization problems [12] is used in order to solve the NLDOP.

The rest of the paper is organized as follows, in Section II the dynamic model of the planar five-bar mechanism is shown. The performance indexes and constraints of the TBIDA are stated in Section III. In the same section, the optimization algorithm to solve the TBIDA is presented. The simulation results of the closed-loop system with the optimal structure control parameters obtained from the TBIDA are shown in Section IV. Finally, conclusions of this approach are summarized in Section V.

2 Dynamic model of the parallel robot

The planar five-bar parallel robot is the manipulator to be designed and it is shown in Fig. 1. It has two degrees of freedom (d.o.f.), $n = 2$. The link lengths are represented by a_1, a_2, a_3, a_4, a_5 , the masses, inertias and mass center distances of the links are represented by $m_1, m_2, m_3, m_4, I_1, I_2, I_3, I_4, l_{c1}, l_{c2}, l_{c3}, l_{c4}$, respectively. $q = [q_1, q_2]^T \in R^2$, $\dot{q} = [\dot{q}_1, \dot{q}_2]^T \in R^2$ are the angular position vector and angular velocity vector of the actuated links respectively, q'_3, q'_4 are the angular positions of the unactuated links, $u = [\tau_1, \tau_2]^T \in R^2$ is the applied generalized force vector and $p = [x_p, y_p]$ is the Cartesian coordinate vector.

The dynamic model is obtained by the reduced model method [14]. This method generates n second-order nonlinear differential equations from the n d.o.f. of the closed chain mechanism, becoming then these equations similar to those of the dynamic equations of serial robots. For the planar five-bar parallel robot it is possible to get two explicit differential equations since the relationship between the dependent and independent generalized coordinates (closed-loop constraints) is an explicit equation.

The dynamic equations of the planar five-bar parallel robot are shown in (1), where

$D(q') = \rho(q')^T D'(q') \rho(q') \in R^{n \times n}$ is a symmetric positive definite inertia matrix, $C(q', \dot{q}') = \rho(q')^T C'(q', \dot{q}') \rho(q') + \rho(q')^T D'(q') \dot{\rho}(q') \in R^{n \times n}$ is the Coriolis and centrifugal matrix, $g(q') = \rho(q')^T g'(q') \in R^n$ is the gravity vector, $u \in R^n$ is the applied generalized force vector. $q' = \sigma(q) \in R^{n'}$ is a dependent generalized coordinate vector (the angular position of the free system q' in terms of the angular position of the constraint system q), where $\sigma(q)$ involves a parameterization of $q' \rightarrow q$. $\dot{q}' = \rho(q') \dot{q}$ is the angular velocity of the free system in terms of the angular velocity of the constraint system $\dot{q}$, where $\rho(q') = \left[\frac{\partial \sigma(q)}{\partial q}, \frac{\partial \sigma(q)}{\partial \dot{q}} \right]$, $\dot{\rho}(q') = \frac{d(\rho(q'))}{dt}$.

$$D(q')\ddot{q}' + C(q', \dot{q}')\dot{q}' + g(q') = u \quad (1)$$

The dynamic equation (1) is valid only (locally) in a compact set which makes it different from regular models of open-chain manipulators. It follows that all control laws for open chain manipulators could also be applied with the only restriction that the guaranteed (Lyapunov) stability conclusions will be local [14].

3 Control-structure design as a NLDOP

In order to state the nonlinear dynamic optimization problem (NLDOP) a control law must be considered. Therefore the closed-loop system must be included as an equality constraint into the NLDOP. The design variables to be optimize are the parameters associated with both the mechanical structure and the controller. This section presents the constraints and the criteria to be considered in order to simultaneously optimize the controller and the structure parameters for executing a specific task.

The planar five-bar parallel robot is the system to be designed. The mathematical model (1) can be written in state variable representation (2), where $x = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T \in R^4$ is the state vector of the system, x_1, x_2 and x_3, x_4 are the positions and velocities of links 1 and 2 respectively, $u = [\tau_1, \tau_2]^T \in R^2$ is the control vector and $p_s \in R^{n_s}$ denotes the structural design parameter vector of the system.

$$\dot{x} = f_s(x, u, p_s) \quad (2)$$

It is well known the conventional PID controller lacks of an asymptotic stability property. Nevertheless, it is widely accepted by industry for automated machines. More than 90% of control loops used in industry use PID controllers [10]. With this in mind, two PID controllers are implemented for the

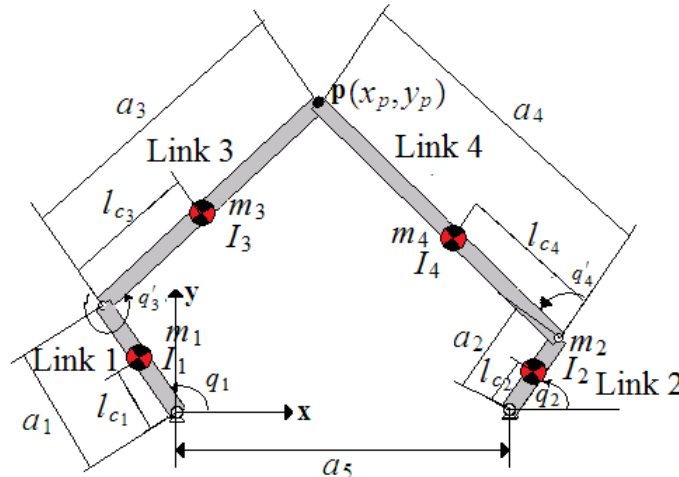


Figure 1: The planar five-bar parallel robot schematic representation.

five-bar mechanism which can be represented by (3), where $e(t) = \bar{q} - x_{1,2} \in R^2$, $\dot{e} = \dot{\bar{q}} - \dot{x}_{3,4} \in R^2$ are the positioning and velocity error vector, $\bar{q}, \dot{\bar{q}} \in R^2$ are the desired positioning and velocity vector, $k_p = [k_{p11}, k_{p22}]$, $k_i = [k_{i11}, k_{i22}]$, $k_d = [k_{d11}, k_{d22}] \in R^{2 \times 2}$ are diagonal positive definite matrices and $p_c = [k_{p11}, k_{i11}, k_{d11}, k_{p22}, k_{i22}, k_{d22}]^T \in R^{n_c}$ is the controller design parameter vector.

$$u = k_p e(t) + k_i \int_0^{t_f} e(t) dt + k_d \dot{e}(t) \quad (3)$$

3.1 Performance criteria

With the lack of an asymptotic stability property for the PID controller, the positioning accuracy is stated as the performance index J_c to be minimized. This index is shown in (4), where $n = 2$ represents the parallel robot d.o.f., $\mu_{c_i} > 0$ are weights assigning relative importance among other performance indexes. $e_i(t) = \bar{q}_i - x_i$ is the position error between the angular position x_i and the desired position $\bar{q}_i$; $e_{i \max}$ represents the maximum error $e_i(t)$ in the time interval $[0, t_f]$ and t_f is the final time

$$J_c = \sum_{i=1}^n \mu_{c_i} \frac{\int_0^{t_f} e_i^2(t) dt}{e_{i \max}^2 t_f} \quad (4)$$

Typically, the structural performance index is based on kinematic considerations [6]. This is not appropriate since the manipulator dynamics are not considered. Yoshikawa [9] introduced the concept of dynamic manipulability that depends on the Jacobian and the inertia matrix of the manipulator. This concept provides a geometric representation and a measure of the ability of a robot for performing end effector accelerations along any task space direction for a set of joint torques. A measure of the dynamic manipulability is the volume of the dynamic manipulability ellipsoid [9]. The task based kinematic design proposed by Kim [7] was the first one, to the best of our knowledge, in utilizing the dynamic manipulability measure as the performance index to be optimized. The link lengths of a parallelogram five-bar planar parallel robot were found by maximizing the dynamic manipulability measure. Nevertheless, the author considered a kinematic design optimization problem, that is, the constraint concerning the closed-loop nonlinear dynamic system behavior was not considered.

In this paper the dynamic manipulability measure is used as the other performance index J_m (5) to be optimized.

$$\begin{aligned} J_m &= \mu_m \frac{M}{\Psi} \\ &= \mu_m \frac{\int_0^{t_f} \sqrt[n]{\det(J(D^T D)^{-1} J^T)} dt}{\int_0^{t_f} \frac{\text{tr}(J(D^T D)^{-1} J^T)}{n_J} dt} \\ &= \mu_m \frac{\int_0^{t_f} \sqrt[n]{\lambda_1 \lambda_2 \cdots \lambda_{n_J}} dt}{\int_0^{t_f} \frac{(\lambda_1 + \lambda_2 + \cdots + \lambda_{n_J})}{n_J} dt}, \end{aligned} \quad (5)$$

where $\mu_m > 0$ is the weight which assigns relative importance among other performance indexes, t_0 and t_f are the initial and final time. $\det(\bullet)$ and $\text{tr}(\bullet)$ are the determinant and trace of the matrix $(\bullet)$ respectively, D is the inertia matrix. J is the Jacobian matrix (6) developed in [15], n_J represents the rank of J . λ_i represents an eigenvalue of $J(D^T D)^{-1} J^T$, M is the dynamic manipulability measure, Ψ represents the arithmetic mean average of the eigenvalues of $J(D^T D)^{-1} J^T$.

$$J = J_q^{-1} J_p, \quad (6)$$

where:

$$\begin{aligned}
J_p &= \begin{bmatrix} (x_p - a_1 \cos q_1) & (y_p - a_1 \sin q_1) \\ (x_p - a_5 - a_2 \cos q_2) & (y_p - a_2 \sin q_2) \end{bmatrix} \\
J_q &= \begin{bmatrix} J_{q11} & 0 \\ 0 & J_{q22} \end{bmatrix} \\
J_{q11} &= (y_p \cos q_1 - x_p \sin q_1) a_1 \\
J_{q22} &= (y_p \cos q_2 + (a_5 - x_p) \sin q_2) a_2
\end{aligned}$$

It is important to note that J_m and J_c are dimensionless and their magnitudes are in the interval $[0, 1]$.

3.2 Constraints

The constraints associated with the controller design consist of establishing a torque bound for each actuator in order to avoid actuator saturation. These constraints are shown in (7), where u_j is the applied torque at the motor j , and $u_{j \max}$ is the maximum torque for the j -th motor.

$$|u_j| \leq u_{j \max} \quad \text{for } j = 1 \text{ to } n \quad (7)$$

The five-bar Grashof criteria [16] are the constraints associated with the structural design and they are shown in (8). These criteria state that the planar five-bar parallel robot is a double-crank linkage if these criteria are fulfilled. Some kinds of inverse and forward kinematic singularities [15] are avoided by the fulfillment of Grashof criteria.

$$\begin{aligned}
a_5 + a_1 + a_2 - a_3 - a_4 &\leq 0 \\
a_1 + a_2 - a_3 &\leq 0 \\
a_1 + a_2 - a_4 &\leq 0 \\
a_3 - a_5 &\leq 0 \\
a_4 - a_5 &\leq 0 \\
a_5 - a_{5\max} &\leq 0 \\
-a_1 &< 0 \\
-a_2 &< 0
\end{aligned} \quad (8)$$

The link lengths of the planar five-bar parallel robot are represented by a_i for $i = 1, \dots, 5$, $a_{5\max}$ is the maximum length of a_5 .

Other important constraint must include the temporary behavior of the closed-loop nonlinear dynamic system in all the time interval $[0, t_f]$. This constraint can be expressed as follows:

$$\dot{x} = f_{sc} \left(x, \bar{q}, \dot{\bar{q}}, p_c, p_s \right), \quad (9)$$

where f_{sc} is the nonlinear system (2) with the feedback controller (3).

3.3 Task based integrated design approach

The purpose of the TBIDA is to find the optimal structure $p_s = [a_1, a_2, a_3, a_4, a_5]^T$ and controller $p_c = [k_{p11}, k_{i11}, k_{d11}, k_{p22}, k_{i22}, k_{d22}]^T$ design parameters of the planar five-bar parallel robot and its controller, that simultaneously minimize the positioning error (4) and maximize the dynamic manipulability measure (5) in order to execute a given task, subject to the constraints at the motor applied

torque (7), the Grashof criteria (8) and the dynamic behavior of the closed-loop nonlinear dynamic system (9).

A weighted-sum approach, is used for handling both the structure and control objectives into a single objective. Then the TBIDA optimization problem can be formulated as follows:

$$\underset{p_s, p_c}{Max} (J_m - J_c), \quad (10)$$

subject to:

- 1.- The dynamic behavior of the closed-loop nonlinear dynamic system (9).
- 2.- The torque limits of the motor (7).
- 3.- The Grashof criteria (8).

A straight line in the Cartesian space is given as the task specification to be followed by the end-effector of the planar five-bar parallel robot. This trajectory is shown in (11), where $c_x = 0.15(m)$, $c_y = 0.15(m)$ are the initial coordinates in the Cartesian space and t is the time.

$$\begin{aligned} x_p &= c_x + 0.03t \\ \dot{x}_p &= 0.03 \\ y_p &= c_y + 0.03t \\ \dot{y}_p &= 0.03 \end{aligned} \quad (11)$$

The inverse kinematic (IK) is required in order to transform the trajectory from the Cartesian coordinates (11) to joint coordinates. For the planar five-bar parallel robot the IK is developed in [15].

3.4 Optimization method

The problem shown in (10) is a nonlinear one which requires a transcription method for converting it from an infinite-dimensional problem to a finite-dimensional approximation in order to use nonlinear programming techniques [17] to solve it. These techniques require to compute both the gradient vector of the performance index with respect to the design variables and the sensitivity vector of the dynamic system states with respect to the same variables. When these techniques deal with nonlinear optimization problems as in (10), they can not guarantee convergence to the global optimum. It is precisely in nonlinear dynamic optimization problems when the use of evolutionary algorithms (EAs) results particularly useful and highly competitive since EAs do not require the transcribing method, the gradient vector and the sensitivity vector. Additionally, being population-based techniques, EAs are less prone to get trapped in local optima, even when dealing with fairly large and complex search spaces.

The EAs can be considered as a broad class of stochastic optimization techniques motivated by the process of natural evolution found in biological organisms. This process can be explained as follows [11]: Reproduction; a way to create new individuals from a current set of them. Mutation; small changes in the genetic codification of individuals which cause significant changes in their features. Competition; a mechanism to measure how fit is an individual to survive in an specific environment. Selection; leads to the maintenance or increase of populations fitness, where fitness is defined as the ability to survive and reproduce in an specific environment.

In this paper, a differential evolution (DE) strategy which is an EA, is used in order to effectively solve the TBIDA optimization problem. DE uses real coding of floating point numbers. The key parameters of the DE strategy are: NP - the population size, CR - the crossover constant, F - the weight applied to random differential (scaling factor), G_{max} - the maximum number of generations (iterations) in order

to finish the DE strategy. The crucial idea behind DE is a scheme for generating trial parameter vectors. Basically, DE adds the weighted difference between two population vectors to a third vector. It is often difficult to choose the values of NP , F , CR since it depends on the specific problem. However, some general guidelines are available in [11].

4 TBIDA optimization results

The TBIDA optimization results are found by using the constraint handling differential evolution (CHDE) algorithm [12] with a population size $NP = 100$ and a maximum number of generation $G_{max} = 3000$. The scaling factor F takes a different value at each generation in the interval $0.3 \leq F \leq 0.9$ and the crossover constant CR is in the interval $0.8 \leq CR \leq 1$ at each optimization process. In this integrated design, it is assumed that each link has a rectangular form with a width of $wd = 0.03(m)$. Other assumption is that masses and mass center distances of the links have the following values: $m_1 = 0.2168(kg)$, $m_2 = 0.1445(kg)$, $m_3 = 0.5059(kg)$, $m_4 = 0.5782(kg)$, $l_{c1} = 0.075(m)$, $l_{c2} = 0.05(m)$, $l_{c3} = 0.175(m)$, $l_{c4} = 0.2(m)$. The inertia moments are computed by $I_i = \frac{m_i(a_i^2 + wd^2)}{12} (Kg m^2)$ for $i = 1 \dots 4$. These inertia moments will be different when the link lengths a_i are changing by the optimization process. The final time is $t_f = 5(s)$.

As stated in the previous section, the trajectory tracking of a straight line (11) is the given task to be executed.

On the other hand, the constant parameters associated with the constraints (7) and (8) are selected as: $a_{5_{max}} = 0.5(m)$, $u_{1_{max}} = u_{2_{max}} = 5(Nm)$. The weights in (10) are chosen as $\mu_{c1} = \mu_{c2} = 1$ and $\mu_m = 1$ to give equal importance in achieving the TBIDA objectives. By selecting these weights, the CHDE algorithm searches for optimal parameters p_c^* and p_s^* . The initial conditions for positions and velocities of the planar five-bar parallel robot are the following: $x_{1_{ini}} = 3.4906(rad)$, $x_{2_{ini}} = 0(rad)$, $x_{3_{ini}} = 0(\frac{rad}{s})$, $x_{4_{ini}} = 0(\frac{rad}{s})$.

The optimal control structure parameters are shown in Table 1. It is important to note that the Grashof criteria (8) are satisfied. The CHDE algorithm converges to the same optimal control structure parameters with different population size, scaling factor F and crossover constants CR .

A comparison between the desired behavior and the closed-loop system behavior in the joint space with optimum control structure parameters is shown in Fig. 2. The trajectory tracking in the joint space presents a good performance. The applied torques at the actuated joints with the optimum control structure parameters are shown in Fig. 3. Note that the torque constraints (7) are fulfilled at all time.

Fig. 4 shows the trajectory tracking of the straight line in the Cartesian space with the control structure parameters obtained by the TBIDA. The Cartesian position error is smaller than $0.0001(m)$. This error could be decreased by increasing the magnitude of the weight μ_{ci} . Because the performance index of the TBIDA optimization is formulated as a weighted-sum approach, the CHDE algorithm finds an unique solution that depends on the selection of the weights μ_{ci} and μ_m . Note that at $t = t_0$, the end-effector of the planar five-bar parallel robot with optimum link lengths is near to the desired straight line trajectory.

Table 1: Control structure optimum parameters with the TBIDA.

	Optimum Parameters
p_c^*	$[21.7772 \ 99.7473 \ 0.4907 \ 43.0841 \ 99.4772 \ 0.6851]^T$
$p_s^*(m)$	$[0.1199 \ 0.1202 \ 0.3162 \ 0.3167 \ 0.3168]^T$
$J_m - J_c$	0.8890

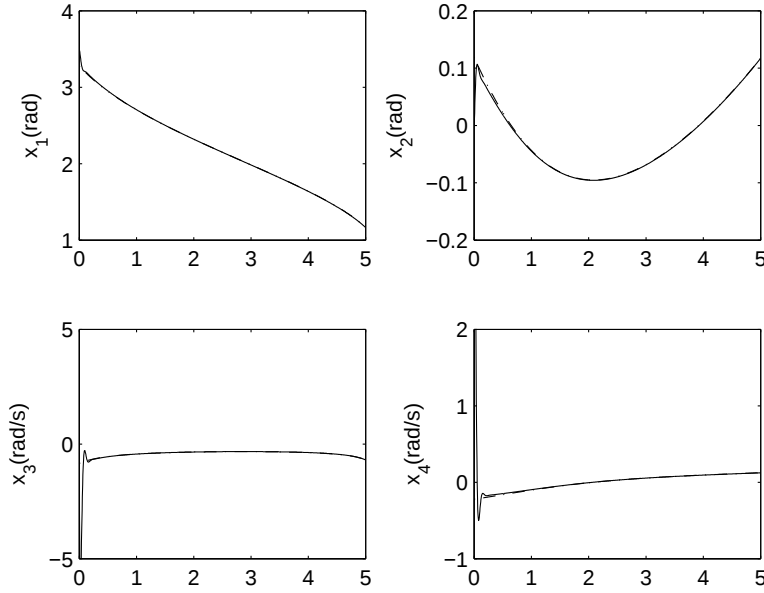


Figure 2: Desired reference (dashed line) and dynamic system behavior (continuous line) in the joint space with the optimal control structure parameters.

Then it requires a minimum motor torque to reach the desired trajectory.

5 Conclusion

In this paper, a task based integrated design approach for the optimal design of a planar five-bar parallel robot is developed. This approach is stated as a nonlinear dynamic optimization problems in order to find the PID controller gains and the link lengths of the parallel robot that maximize the dynamic manipulability (a dexterity measure) and minimize the position error in a simultaneous way. This approach considers the constraints in the applied motor torque, the Grashof criteria and the nonlinear dynamic system behavior.

Although the PID controller for trajectory tracking problems does not guarantee asymptotic convergence of the error to zero, with the TBIDA the error tends to zero since a practical point of view.

In this approach, a task to be executed by the end-effector of a parallel robot is given by the designer. This approach finds an optimum selection of the link lengths and controller gains in order to satisfy a structure and a control criterium. The problems associated with the parallel robot design and the controller design for trajectory tracking are solved by the TBIDA. The performance in the positioning errors, dexterity and applied torque for a given task is improved with the proposed approach.

Future work will involve the experimental results of the TBIDA. However, it will be necessary to implement another optimization process in order to adjust the optimal structure parameters to a geometry of the links, as m_i and l_{ci} will remain constant as considered above.

The CHDE algorithm presents the same convergence at the optimum parameters with different initial conditions of the algorithm. This solution depends on the given weights for each performance index. Future work will include multicriteria optimization [18] in order to find a set of all possible solutions, called a Pareto optimal set [18] and optimal selection of all link parameters. Hence the optimum solution will depend on the decision maker.

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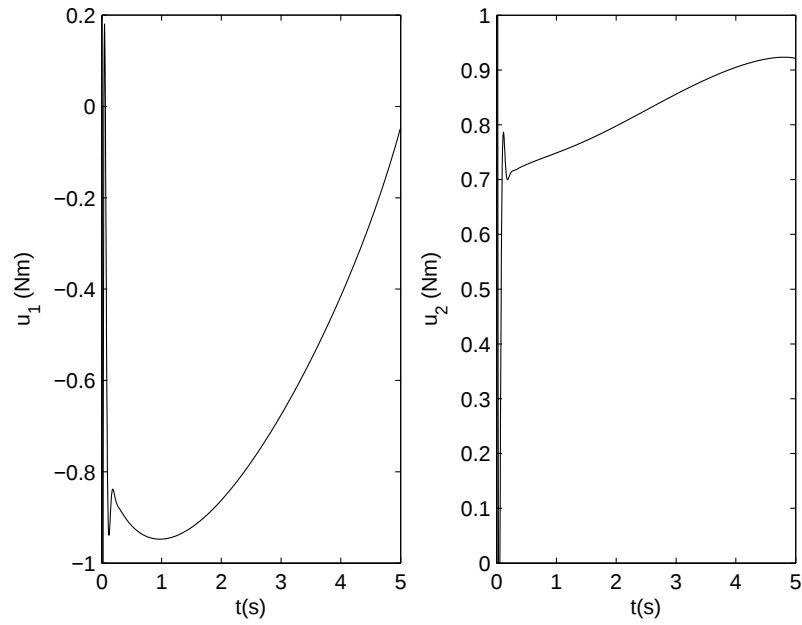


Figure 3: The input torques in the dynamic system with the optimal control structure parameters.

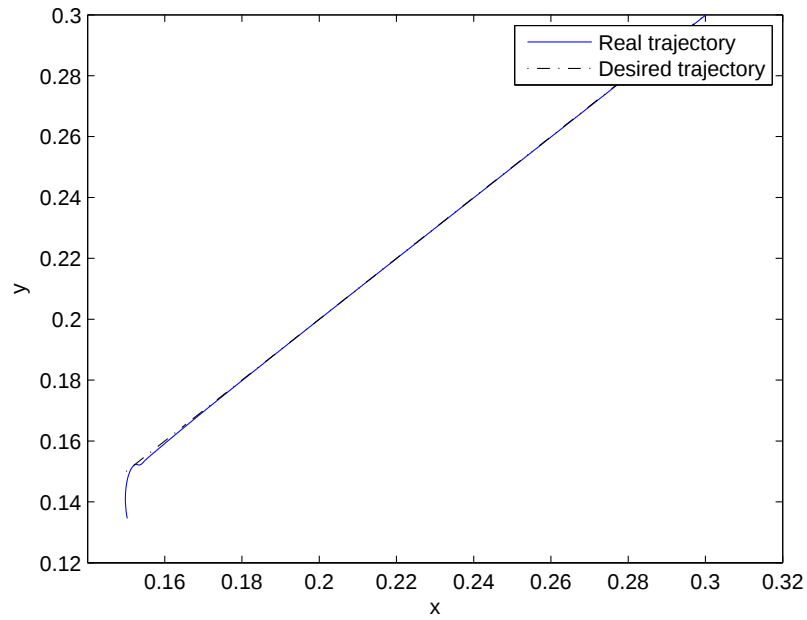


Figure 4: Desired and real end-effector trajectory tracking in the Cartesian space with the optimal control structure parameters.